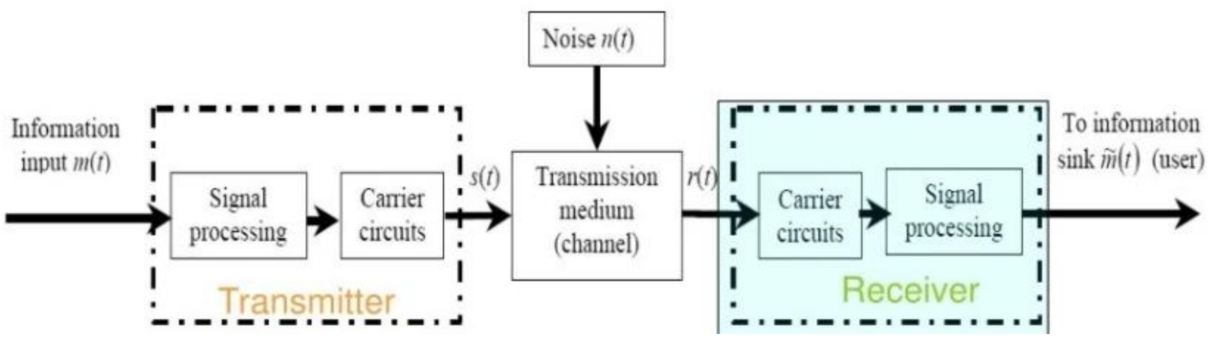
Course Content:

۱. What is communication
۲. Basic elements of a communication system
۳. Different Sources of a Communication system
٤. Signals and Channels
٥. Modulation and Demodulation
٦. Amplitude Modulation (AM)
٧. Frequency Modulation (FM)
٨. Channel Noise Analysis

۱. About the Communication & Basic elements



1 . بلوک دیاگرام یک سیستم مخابراتی

Radio Spectrum and Frequency bands

Notes

a- ratio frequency = 3KHz $\rightarrow$ 300GHz and none of the frequency above 300GHz is classified as radio waves.

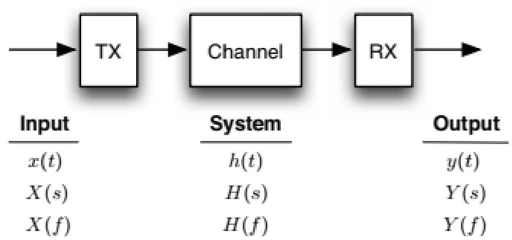
b- Sound velocity = 345 m/s .

c- Human voice 20 $\rightarrow$ 4000 Hz

d- Audible rang 20 $\rightarrow$ 20 000 Hz

Frequency	Band
10 kHz to 30 kHz	Very Low Frequency (VLF)
30 kHz to 300 kHz	Low Frequency (LF)
300 kHz to 3 MHz	Medium Frequency (MF)
3 MHz to 30 MHz	High Frequency (HF)
30 MHz to 144 MHz 144 MHz to 174 MHz 174 MHz to 328.6 MHz	Very High Frequency (VHF)
328.6 MHz to 450 MHz 450 MHz to 470 MHz 470 MHz to 806 MHz 806 MHz to 960 MHz 960 MHz to 2.3 GHz 2.3 GHz to 2.9 GHz	Ultra High Frequency (UHF)
2.9 GHz to 30 GHz	Super High Frequency (SHF)
30 GHz and above	Extremely High Frequency (EHF)

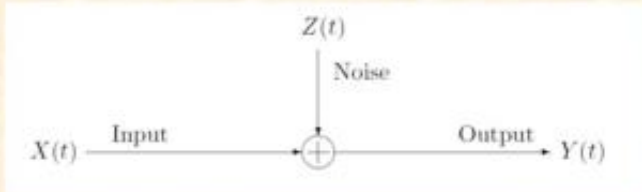
٢. Signals, messages and channels



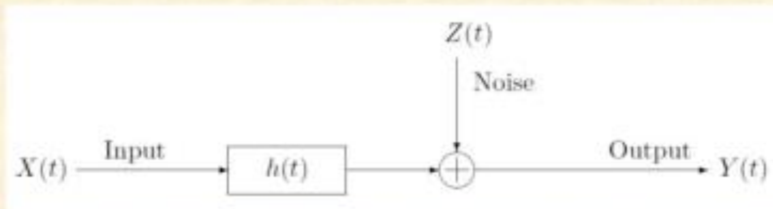
1. Linear Time Invariant System (LTI System)
2. Linear Time Variant System (LTV System)
3. Continuous and Discrete time system
4. Causal and Non Causal Systems

Channel Model

- AWGN channel



- Linear channel



Duplex Transmission

- Simplex
  - one-way transmission only, e.g. broadcast systems
- Half-duplex
  - two-way transmission, but the common channel is alternately used for transmission in opposite direction, e.g. interphone systems installed on taxies
- Full-duplex
  - simultaneous two-way transmission, e.g. public telephone systems

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- **Simplex (SX) –** one direction only, e.g. TV
- **Half Duplex (HDX) –** both directions but not at the same time, e.g. CB radio
- **Full Duplex (FDX) –** transmit and receive simultaneously between two stations, e.g. standard telephone system.

The channel (regardless of the type) degrades the transmitted signal in a number of ways :-

1-Attenuation: it is defined as a reduction of signal strength. (increase with channel length).

2-Distortion: is waveform perturbation caused by imperfect response of the system to the desired signal itself. Distortion may be corrected, or at least reduced with the help of special filters called equalizers.

3-Interference: is contamination by extraneous signals from human source, other transmitters, machinery, switching circuits, ……,

Filtering removes interference.

4-Noise: it is a random and undesirable signal from external and internal causes.It is defined as the ratio of the signal power to the noise power:

Signal analysis (Time and Frequency)

Classification of signals:

1. Continuous & Discrete Signals
2. Randam & Diterminstic Signals
3. Periodic & Non Periodic Signals
4. Causal & Non Causal Signals
5. Energy & Power signals
6. Even & Odd signals

complex-valued signal $g(t)$

$$E_g = \int_{\infty}^{\infty} |g(t)|^2 dt. \qquad |g(t)|^2 = g(t)g^*(t).$$

If $g(t)$ is real-valued then $g(t) = g^*(t)$. Thus,

$$|g(t)|^2 = g(t)g^*(t) = g^2(t).$$

$$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt.$$

Periodic Signals: The waveform $g(t)$ is periodic with period T_0 if $g(t) = g(t + kT_0)$, for all t and $k \in \mathcal{Z}$ (the set of all integers: $0, \pm 1, \pm 2, \ldots$).

Energy Signals: The waveform $g(t)$ is an energy signal if $0 \leq E_g < \infty$.

Power Signals: The waveform $g(t)$ is a power signal if $0 \leq P_g < \infty$. Please note that a signal cannot simultaneously be both an energy and a power signal.

Time Scaling: $g(t) \longrightarrow g(\alpha t)$ with $\alpha \in \mathcal{R}$. If $\alpha > 1$, then $g(\alpha t)$ represents a time-compressed (by a factor of α) version of $g(t)$, whereas if $\alpha < 1$, then $g(\alpha t)$ represents a time-stretched (by a factor of α) version of $g(t)$:

Time Reversal: $g(t) \longrightarrow g(-t)$ represents a mirror image of $g(t)$ with respect to the vertical axis $t = 0$. Similarly, $g(t_0 - t)$ with $t_0 \in \mathcal{R}$ first takes the mirror image of $g(t)$ with respect to the vertical axis and then shifts the resulting waveform by t_0 units.

The unit impulse signal $\delta(t)$ is defined as:

$$\delta(t) = 0, \text{ if } t \neq 0; \quad \text{and} \quad \int_{-\infty}^{\infty} \delta(t)dt = 1$$

For an arbitrary function $x(t)$ and constant $K \in \Re$ we have:

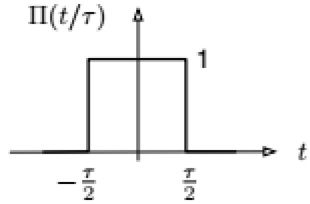
$$\begin{aligned}
&\int \delta(t)x(t)dt = x(0); \\
&\int \delta(t - t_0)x(t)dt = x(t_0), \quad \text{(the } \textit{sifting} \text{ property)}; \\
&\int K\delta(t)dt = K.
\end{aligned}$$

Convolution with the δ-function:

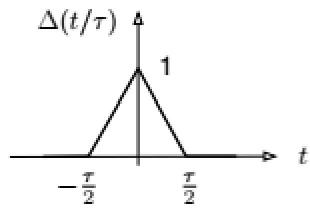
$$\begin{aligned}
\delta(t - t_0) * x(t) &= \int \delta(\lambda - t_0)x(t - \lambda)d\lambda \\
&= x(t - \lambda)|_{\lambda = t_0}, \\
&= x(t - t_0).
\end{aligned}$$

unit rectangular pulse function

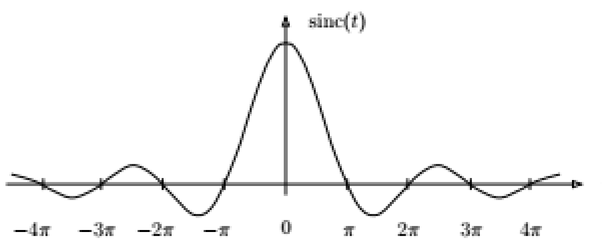
$$\Pi(t) = \begin{cases} 1, & |t| < \frac{1}{2}; \\ \frac{1}{2}, & |t| = \frac{1}{2}; \\ 0, & |t| > \frac{1}{2}. \end{cases}$$



$$\Delta(t) = \begin{cases} 1 - 2|t|, & |t| \leq \frac{1}{2}; \\ 0, & |t| > \frac{1}{2}. \end{cases}$$



$$\text{sinc}(t) = \frac{\sin(t)}{t}.$$



Signal Frequency Analysis

Given a periodic waveform $x_p(t)$ with period T_0

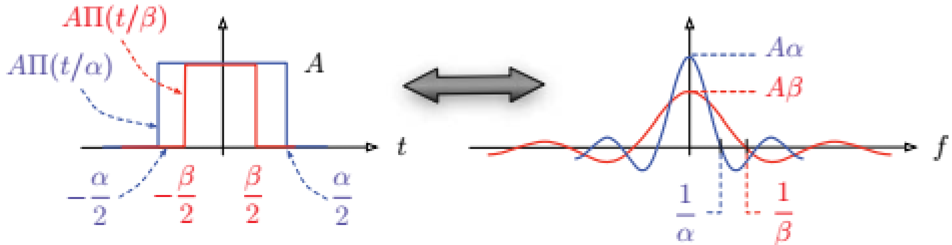
$$x_p(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} \qquad x_p(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t),$$

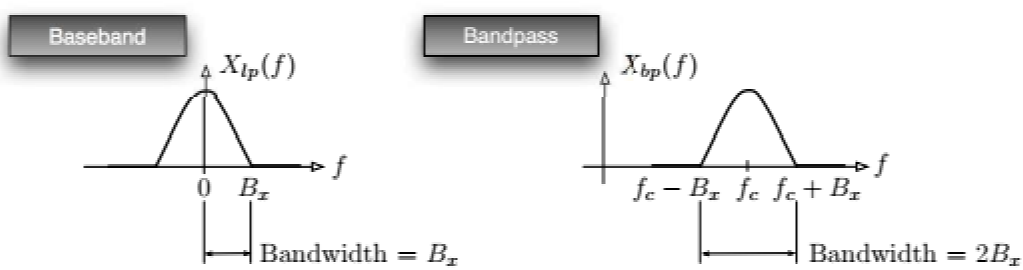
$$D_n = \frac{1}{T_0} \int_{T_0} x_p(t) e^{-jn\omega_0 t} dt.$$

$$
\begin{aligned}
a_0 &= \frac{1}{T_0}\int_{T_0} x_p(t)dt, \\
a_n &= \frac{2}{T_0}\int_{T_0} x_p(t)\cos n\omega_0 t\,dt, \quad n = 1, 2, \ldots \\
b_n &= \frac{2}{T_0}\int_{T_0} x_p(t)\sin n\omega_0 t\,dt, \quad n = 1, 2, \ldots
\end{aligned}
$$

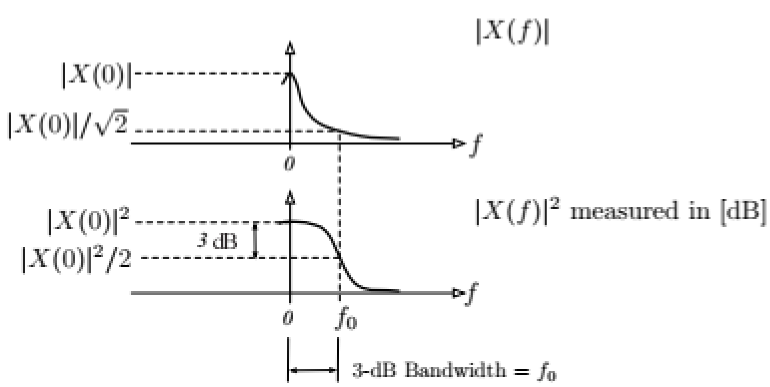
Fourier Transform

$$
\begin{aligned}
X(f) &= \mathcal{F}[\,x(t)\,], \\
&= \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt,
\end{aligned}
\qquad\qquad
\begin{aligned}
x(t) &= \mathcal{F}^{-1}[\,x(t)\,], \\
&= \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df.
\end{aligned}
$$





the measurement of the 3-dB bandwidth for baseband signals.



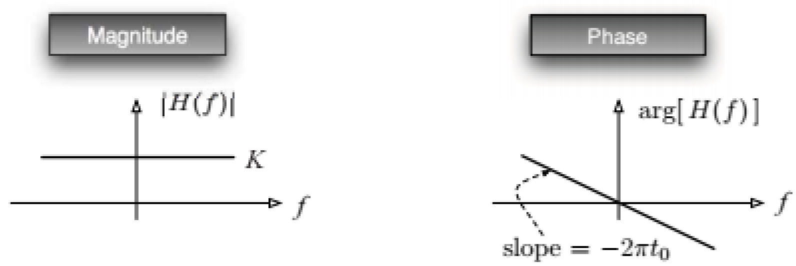
A **distortionless system** with input $x(t)$ and $y(t)$ satisfies the input-output relation:

$$y(t) = K\,x(t - t_0), \tag{3.56}$$

$$Y(f) = H(f)X(f),$$

$$\begin{aligned} Y(f) &= \mathcal{F}[\,y(t)\,], \\ &= \mathcal{F}[\,Kx(t - t_0)\,], \\ &= K\,\mathcal{F}[\,x(t - t_0)\,], \\ &= KX(f)e^{-j2\pi f t_0}, \end{aligned}$$

$$H(f) = \frac{Y(f)}{X(f)} = K\,e^{-j2\pi f t_0}.$$



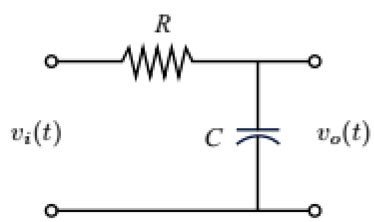


Figure 3.26: The RC-lowpass system.

$$H_{rc}(f) = \frac{1}{1 + j2\pi f RC}$$

$$|H_{rc}(f)| = \frac{1}{\sqrt{1 + (2\pi f RC)^2}}, \quad \text{and} \quad \arg[H_{rc}(f)] = -\tan^{-1}(2\pi f RC).$$

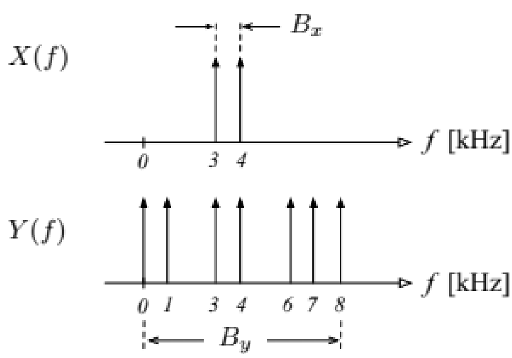
$$|H_{rc}(f_0)|^2 = \frac{|H_{rc}(0)|^2}{2} \quad \Longrightarrow \frac{1}{2} = \frac{1}{1 + (2\pi f_0 RC)^2} \quad \Longrightarrow f_0 = \frac{1}{2\pi RC}.$$

Example 3.6: Let $x(t) = A\cos\omega_1 t + B\cos\omega_2 t$ with

$$y(t) = g[x(t)] = x(t) + 2x^2(t).$$

$$\begin{aligned} y(t) &= \left[\, A\cos\omega_1 t + B\cos\omega_2 t \,\right] + 2\left[A\cos\omega_1 t + B\cos\omega_2 t\right]^2, \\ &= A\cos\omega_1 t + B\cos\omega_2 t + 2A^2\cos^2\omega_1 t + 2B^2\cos\omega_2 t + 4AB\cos\omega_1 t\cos\omega_2 t, \\ &= A\cos\omega_1 t + B\cos\omega_2 t + A^2 + A^2\cos 2\omega_1 t + B^2 + B^2\cos 2\omega_2 t \\ &\quad + 2AB\cos(\omega_1+\omega_2)t + 2AB\cos(\omega_1-\omega_2)t. \end{aligned}$$

If $f_1 = 4$ kHz and $f_2 = 3$ kHz, the input and output spectra will be as shown: Observe that



Parseval's Theorem

$$\int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega$$

$$P = |v(t)|^2 = \frac{1}{T_o}\int_{T_o} |v(t)|^2 dt$$

$$\therefore\ P = \sum_{n=-\infty}^{\infty} |Cn|^2$$

Example:

If Cn=[1 3-j4 2+j 2 2-j 3+j4 1], find P ,an energy, the instantaneous power.

Solution: $P = 1^2 + (3^2+4^2) + (2^2+1^2) + 2^2 + (2^2+1^2) + (3^2+4^2) + 1^2 = 66 wat.$

Energy=P.T T: period

$$P = v^2(t)/R$$

٣. Modulation % Demodulation

In modulation, a <u>message</u> signal, which contains the <u>information</u> is used to control the parameters of a <u>carrier</u> signal, so as to impress the information onto the carrier.

<u>The Messages</u>

The message or modulating signal may be either:

analogue – denoted by *m*(*t*)

digital – denoted by *d*(*t*) – *i.e.* sequences of 1's and 0's

The message signal could also be a multilevel signal, rather than binary; this is not considered further at this stage.

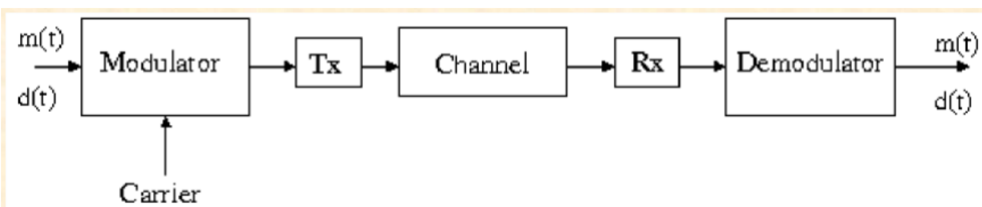
<u>The Carrier</u>

The carrier could be a 'sine wave' or a 'pulse train'.

<u>Consider a 'sine wave' carrier:</u>

$$v_c(t) = V_c \cos(\omega_c t + \varphi_c)$$

- If the message signal *m*(*t*) controls amplitude – gives AMPLITUDE MODULATION AM
- If the message signal *m*(*t*) controls frequency – gives FREQUENCY MODULATION FM
- If the message signal *m*(*t*) controls phase- gives PHASE MODULATION PM or ϕM



Need for Modulation:

- Baseband signals are incompatible for direct transmission over the medium so, modulation is used to convey (baseband) signals from one place to another.
- Allows frequency translation:
  - Frequency Multiplexing
  - Reduce the antenna height
  - Avoids mixing of signals
  - Narrowbanding
- Efficient transmission
- Reduced noise and interference

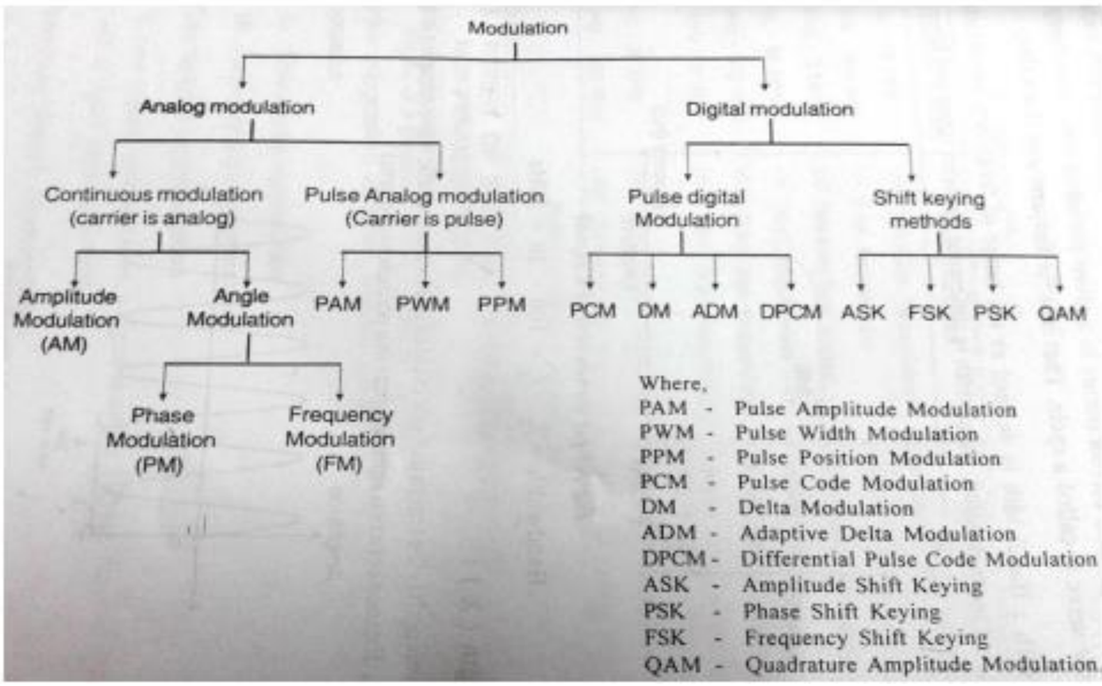
The following are the major points for which modulation is essential, those are

Practicality of Antenna

Reduction of Interference

Reduction of Noise

Multiplexing



٤. Analogue, Amplitude Modulation (AM)

Example: Double sideband with carrier (DSB-WC), Double- sideband suppressed carrier (DSB-SC), Single sideband suppressed carrier (SSB-SC), vestigial sideband (VSB)

- **Angle modulation (frequency modulationλ & phase modulation)**
  Example: Narrow band frequency modulation (NBFM), Widebandλ frequency modulation (WBFM), Narrowband phase modulation (NBPM), Wideband phase modulation (NBPM)

Pulse Modulation

- Carrier is a train of pulses
- Example: Pulse Amplitude Modulation (PAM), Pulse width modulation (PWM) Pulse Position Modulation (PPM)

Amplitude Modulation system

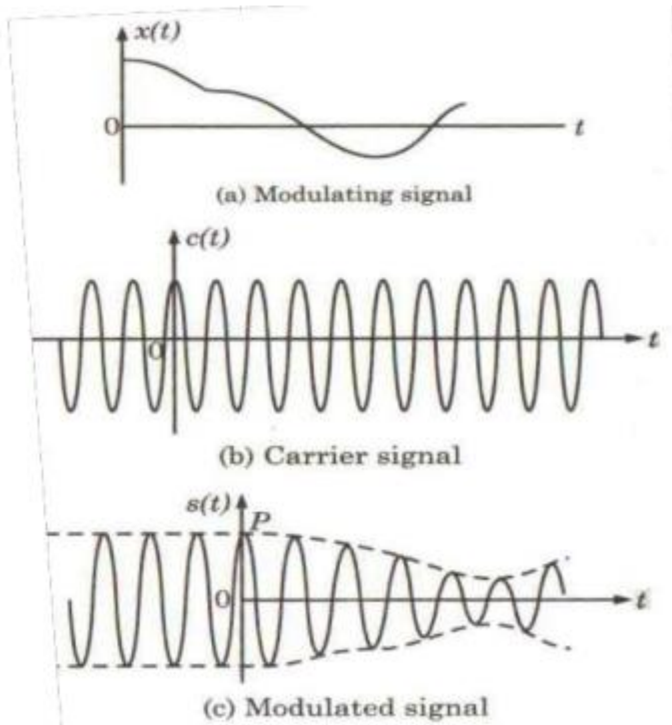


Illustration of Amplitude Modulation.

Single Tone Modulation:

$$m(t) = A_m \cos(2\pi f_m t)$$

$$s(t) = A_c [1 + m\cos(2\pi f_m t)] Cos(2\pi f_c t)$$

Let A_{max} and A_{min} denote the maximum and minimum values of the envelope of the modulated wave. Then from the above equation (2.12), we get

$$\frac{A_{max}}{A_{min}} = \frac{A_c(1+m)}{A_c(1-m)}$$

$$m = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

$$s(t) = A_c \cos(2\pi f_c t) + \frac{1}{2} m A_c \cos[2\pi(f_c + f_m)t] + \frac{1}{2} m A_c \cos[2\pi(f_c - f_m)t]$$

The Fourier transform of $s(t)$ is obtained as follows.

$$s(f) = \frac{1}{2} A_c[\delta(f - f_c) + \delta(f + f_c)] + \frac{1}{4} m A_c[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)]$$

$$+\frac{1}{4} m A_c[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)]$$

Power relations in AM waves:

$$s(t) = A_c Cos(2\pi f_c t) + \frac{1}{2} m A_c Cos[2\pi(f_c + f_m)t] + \frac{1}{2} m A_c Cos[2\pi(f_c - f_m)t]$$

Therefore the total power in the amplitude modulated wave is given by

$$Pt = \frac{V^2{}_{car}}{R} + \frac{V^2{}_{LSB}}{R} + \frac{V^2{}_{USB}}{R} \quad \text{.......................(2)}$$

$$P_C = \frac{V^2{}_{car}}{R} = \frac{\left(A_c/\sqrt{2}\right)^2}{R} = \frac{A_c{}^2}{2R}$$

$$P_{LSB} = \frac{V_{LSB}{}^2}{R} = \left(\frac{mA_c}{2\sqrt{2}}\right)^2 \frac{1}{R} = \frac{m^2 A_c{}^2}{8R} = \frac{m^2}{4} P_c$$

$$P_{USB} = \frac{V_{USB}{}^2}{R} = \left(\frac{mA_c}{2\sqrt{2}}\right)^2 \frac{1}{R} = \frac{m^2 A_c{}^2}{8R} = \frac{m^2}{4} P_c$$

Therefore total average power is given by

$$P_t = P_c + P_{LSB} + P_{USB}$$

$$P_t = P_c + \frac{m^2}{4}P_c + \frac{m^2}{4}P_c$$

$$P_t = P_c\left(1+\frac{m^2}{4}+\frac{m^2}{4}\right)$$

$$P_t = P_c\left(1+\frac{m^2}{2}\right)$$

$$\frac{P_{SB}}{P_t} = \frac{P_c\left(m^2/2\right)}{P_c\left(1+m^2/2\right)}$$

$$\frac{P_{SB}}{P_t} = \frac{m^2}{2+m^2}$$

efficiency of AM system

$m(t)$= Am Cos $w_m t$ $c(t)$= Ac Cos $w_c t$

A= Ac+ m(t)...................................

A= Ac+ Am Cos $w_m t$

A= Ac [1+ (Am/Ac) Cos $w_m t$]

A= Ac[1+ Ma Cos $w_m t$]

Now the modulated signal M(t) can be determined as

M(t) = A Cos $w_c t$

= Ac [1+Ma Cos $w_m t$] Cos $w_c t$

= Ac Cos $w_c t$ + AcMa Cos $w_c t$ Cos $w_m t$

= Ac Cos $w_c t$ + [AcMa/2] [Cos$(w_c+w_m)t$ + Cos $(w_c-w_m)t$]

= Ac Cos $w_c t$ + [AcMa/2] [Cos$(w_c+w_m)t$] + [AcMa/2] [Cos $(w_c-w_m)t$]

Frequency Spectrum of AM wave

Carrier with frequency component w_c

Upper side band with frequency component (w_c+w_m)

Lower side band with frequency component (w_c-w_m)

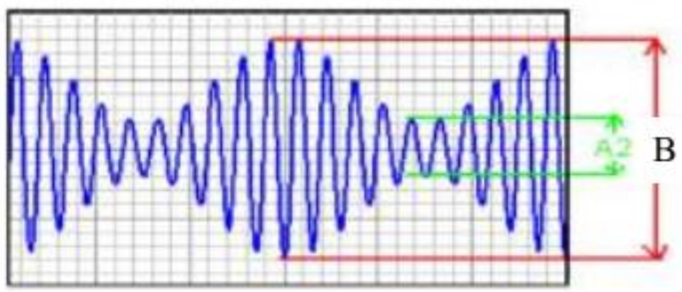
So Band Width

$B = (w_c+w_m)-(w_c-w_m)$

$= w_c + w_m - w_c + w_m$

$= 2\ w_m$

Percent Modulation [m]



From (0 → 100%)
m = signal amplitude / carrier amplitude
m = (B / A) × 100
Modulation index = M . I = B / A

$$m = \frac{E_c(\max) - E_c(\min)}{E_c(\max) + E_c(\min)} \times 100$$

Example: An audio signal given by 15sin2π (2000t) amplitude modulates a sinusoidal carrier wave 60sin2π (100000t), determine:

a) Modulation index.

b) Percent modulation.

c) Frequencies of signal and carrier.

d) Frequency spectrum of modulated wave.

Solution: B = 15 and A = 60

a) $M.I = \frac{B}{A} = \frac{15}{60} = 0.25$

b) m = M.I × 100 = 0.25 × 100 = 25%

c) f_m = 2000 Hz , f_c = 100000 Hz

d) the three frequencies present in the modulated CW are

i- 100000 Hz = 100 KHz

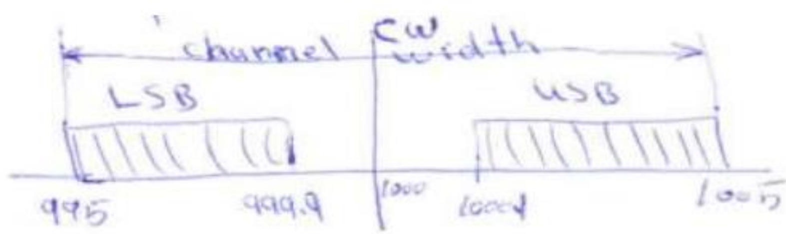
ii- 100000 + 2000 = 102000 KHz

iii- 100000 – 2000 = 98000 Hz = 98 KHz

Example:

In a broadcasting studio, a 1000 KHz carrier is modulated by an audio signal of frequency rang 100 – 5000 Hz, find (i) width or freq. rang of sidebands. (ii) maximum and minimum freq.'s of USB (iii) max. and min. freq.'s of LSB and ,(iv) width of the channel.

Solution:



(i) width of sideband = 5000 – 100 = 4900 Hz

(ii) max. freq. of USB = 1000 + 5 = 1005 KHz

min. freq. of USB = 1000 + 0.1 = 1000.1 KHz

(iii) max. freq. of LSB = 1000 – 0.1 = 999.9 KHz

min. freq. of LSB = 1000 + 5 = 995 KHz

(iv) width of channel = 1005 -995 = 10 KHz

Power Relation in an AM wave

$P_c \alpha A^2 \Rightarrow P_c = KA^2$,K : constant , B=mA

USBpower, $P_{USB} \alpha (\frac{mA}{2})^2 = \frac{KB^2}{4}$

LSB power, $P_{LSB} \alpha (\frac{B}{2})^2 = \frac{KB^2}{4}$

Total sideband power $P_{sb} = 2 \times \frac{KB^2}{4} = \frac{KB^2}{2}$

Total power radiated out from the antenna is

$$P_T = P_c + P_{SB} = KA^2 + \frac{KB^2}{2}$$

$$P_T = KA^2 + \frac{K}{2}(mA)^2 = KA^2(1 + \frac{m^2}{2}), \ldots\ldots\ \ldots\ p_c = KA^2$$

1- $P_T = P_c(1 + \frac{m^2}{2}) = P_c(\frac{2 + m^2}{2})$ If m=1 or 100% $P_T = 1.5P_c$

2- $\therefore\ P_c = P_T\ (\frac{2}{2 + m^2})$ If m=1 , $P_c = \frac{2}{3} P_T$

3- $P_{SB} = P_T - P_c = P_c(1 + \frac{m^2}{2}) - P_c$, if m=1 ,

$$P_{SB} = \frac{m^2}{2} P_c = P_T\ (\frac{m^2}{2 + m^2})$$

$$P_{SB} = \frac{1}{2} P_c$$

4- $P_{USB} = P_{LSB} = \frac{1}{2} P_{SB} = \frac{m^2}{4} P_c = \frac{1}{2}(\frac{m^2}{2 + m^2}) P_T$ If m=1,

$$P_{USB} = P_{LSB} = \frac{1}{4} P_c$$

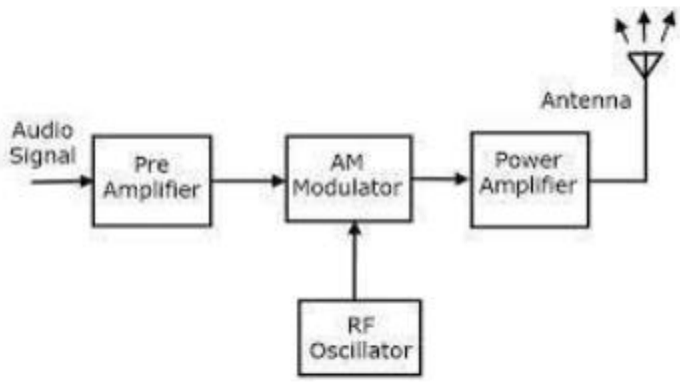
$$P_{USB} = \frac{1}{6} P_T$$

Example: The total power content of an AM wave is 2.64Kw at a modulation factor of 80%, determine the power content of (i) carrier. (ii) Each side-band.

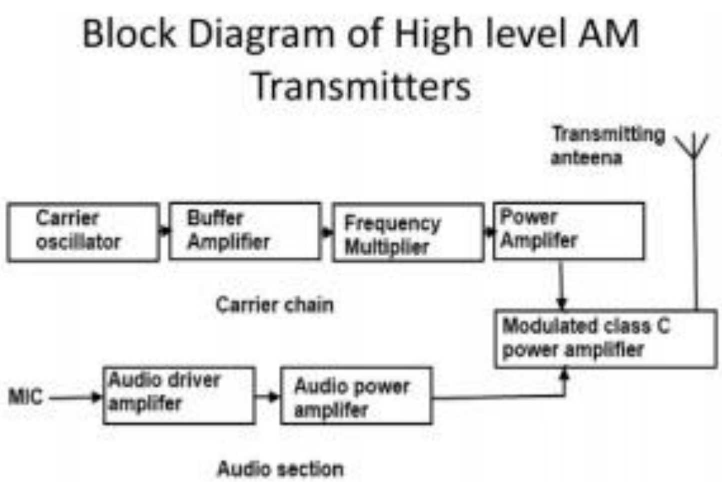
$$P_c = P_T\left(\frac{2}{2+m^2}\right) = 2.64\times10^3\left(\frac{2}{2+0.8^2}\right) = 2000w$$

$$P_{USB} = P_{LSB} = \frac{1}{2}P_{SB} = \frac{m^2}{4}P_c = \frac{0.8^2}{4}\times 2000 = 320w$$

Generation of Amplitude Modulated Signal



Low Level AM Generation



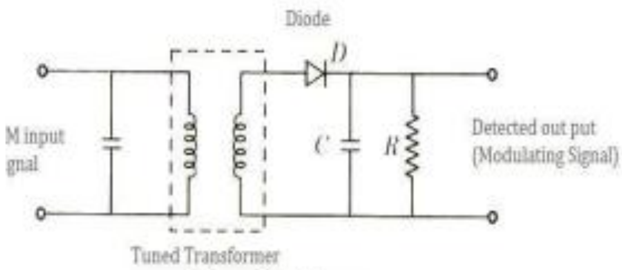
Demodulation of AM waves

Linear the Detector (envelope detector)

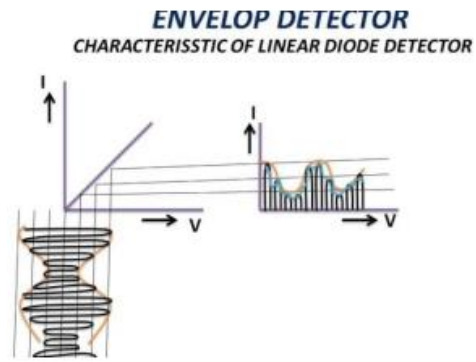
Square Law Detector

Linear the Detector (envelope detector)

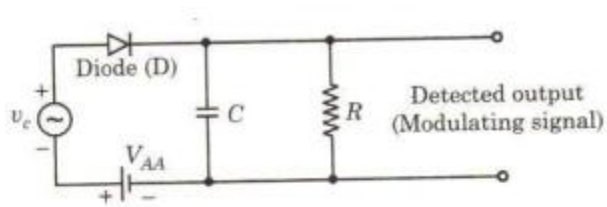
A linear diode detector is popularly used in commercial radio receivers. As it is used to demodulate at a high-level AM signal, the diode used in the demodulator circuit operates over the linear region of its V-I characteristics. The linear diode detector circuit is simple and inexpensive in nature.



A Linear Diode Detecter



Square Law detector



Basic circuit of square law diode detector.

Let V be the voltage of AM signal which can be denoted as

$V = A(1 + Ma \cos w_m t) \cos w_c t$

Let 'I' be the diode current that flows through the load register that provides the overall output.Now according to Square Law principle 'I' be proportional to the square of the input voltage V.

Now $I = aV + bV^2$

By substituting the value of $V = A(1 + Ma \cos w_m t) \cos w_c t$ we get

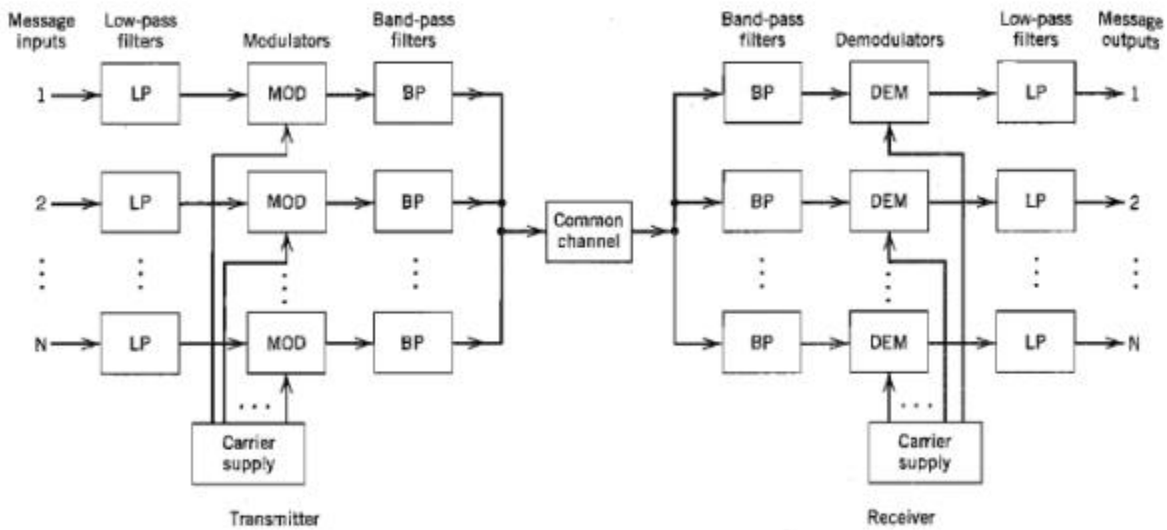
$I = a[A\cos w_c t + AMa\cos w_m t \cos w_c t] + b[A\cos w_c t + AMa\cos w_m t \cos w_c t]^2$

If the above equation is expanded then we may observe the terms having frequency components like $2w_c$, $2(w_c+w_m)$, $2(w_c-w_m)$, w_m, $2w_m$ etc. Now by passing the current through a low pass filter we can able to separate the term having frequency component w_m only.

Frequency Division Multiplexing

Multiplexing is the name given to techniques, which allow more than one message to be transferred via the same communication channel. The channel in this context could be a transmission line, *e.g.* a twisted pair or co-axial cable, a radio system or a fibre optic system *etc*.

FDM is derived from AM techniques in which the signals occupy the same physical 'line' but in different frequency bands. Each signal occupies its own specific band of frequencies all the time, *i.e.* the messages share the channel **bandwidth**.



- Application of AM - Radio broadcasting, TV pictures (video), facsimile transmission
- Frequency range for AM - 535 kHz – 1600 kHz
- Bandwidth - 10 kHz

٥. Frequency Modulation (FM)

Instantaneous Frequency

$x(t) = \cos(\omega_c t + \theta_0)$

The angle of the cosine $\theta(t) = \omega_c t + \theta_0$ is a linear relationship with respect to t

The instantaneous frequency of $y(t)$

$$\omega_i(t) = \frac{d\theta(t)}{dt}.$$

Phase Modulation (PM)

$$g_{PM}(t) = A \cdot \cos\left[\omega_c t + k_p m(t)\right]$$

$$\theta_{PM}(t) = \omega_c t + k_p m(t),$$

$$\omega_i(t) = \omega_c + k_p \frac{dm(t)}{dt} = \omega_c + k_p \dot{m}(t).$$

So, the frequency of a PM signal is proportional to the derivative of the message signal.

Frequency Modulation (FM)

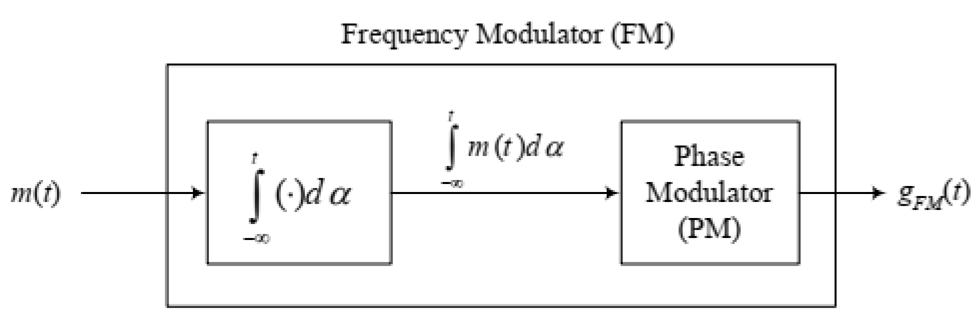
$$g_{FM}(t) = A \cdot \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha) d\alpha\right],$$

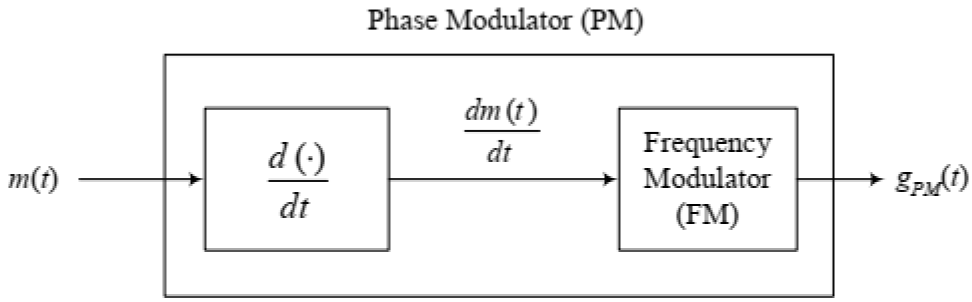
The phase and instantaneous frequency of this FM are

$$\theta_{FM}(t) = \omega_c t + k_f \int_{-\infty}^{t} m(\alpha) d\alpha,$$

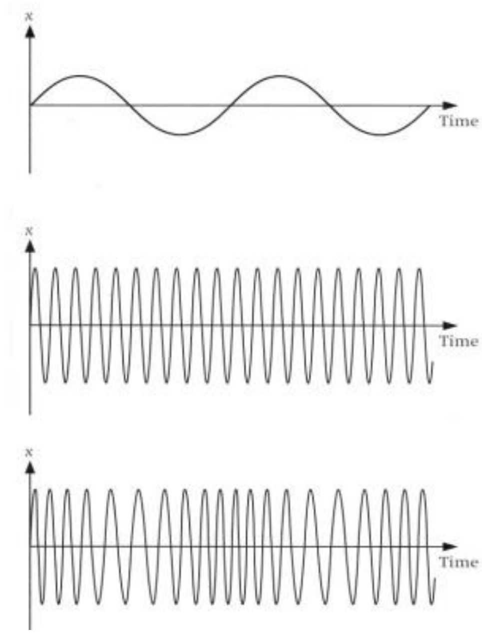
$$\omega_i(t) = \omega_c + k_f \frac{d}{dt}\left[\int_{-\infty}^{t} m(\alpha) d\alpha\right] = \omega_c + k_f m(t).$$

Relation between PM and FM





In **F**requency **M**odulation (**FM**) the instantaneous value of the information signal controls the frequency of the carrier wave. This is illustrated in the following diagrams.

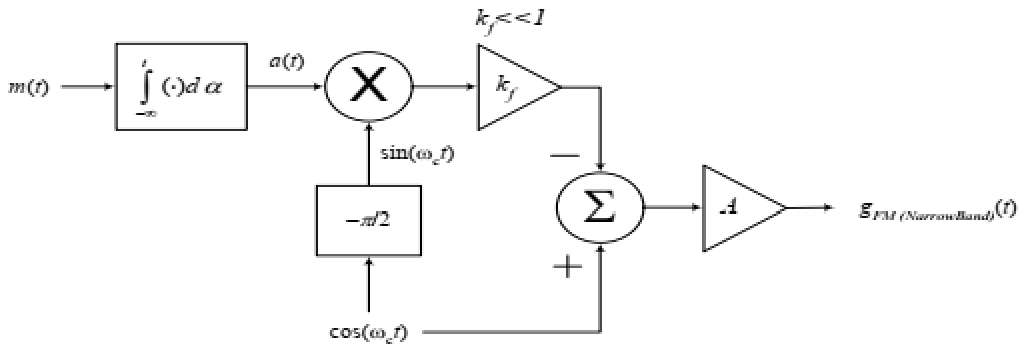


Narrowband FM and PM

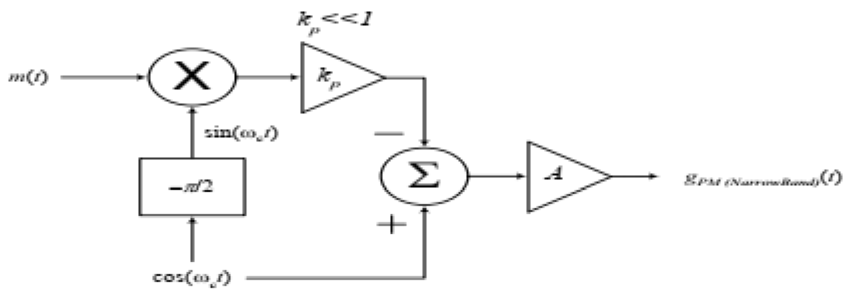
$$g_{FM}(t) = A \cdot \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha) d\alpha\right].$$

A narrow band FM or PM signal satisfies the condition

$$|k_f a(t)| \ll 1$$

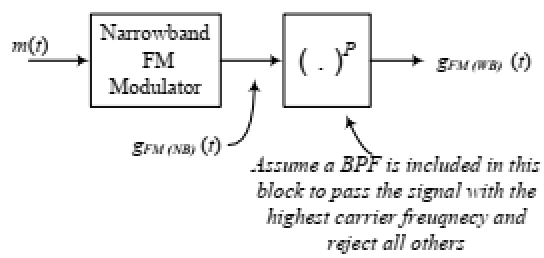


Narrowband FM Modulator



Narrowband PM Modulator

Generation of Wideband FM Signals



SINGLE-TONE FREQUENCY MODULATION

$$n(t) = A_n \cos(2n f_n t)$$

$$f(t) = f_c + k_f A_n \cos(2n f_n t) = f_c + \Delta f \cos(2n f_n t)$$

$$\Delta f = k_f A_n$$

$$\theta(t) = 2\pi f_c t + 2\pi k_f \int_0^t A_m \cos(2\pi f_m t) dt$$

$$= 2\pi f_c t + 2\pi k_f \frac{A_m}{2\pi f_m} \sin(2\pi f_m t)$$

$$= 2\pi f_c t + \frac{k_f A_m}{f_m} \sin(2\pi f_m t)$$

$$= 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t)$$

$$\therefore \theta(t) = 2\pi f_c t + \beta_f \sin(2\pi f_m t)$$

Where

$$\beta_f = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m}$$

Therefore, the single-tone FM wave is

$$s_{FM}(t) = A_c \cos[2\pi f_c t + \beta_f \sin(2\pi f_n t)]$$

single-tone phase modulated wave may be determined from Eq.as

$$s_{PM}(t) = A_c \cos[2\pi f_c t + k_p A_n \cos(2\pi f_n t)]$$

$$\text{or, } s_{PM}(t) = A_c \cos[2\pi f_c t + \beta_p \cos(2\pi f_n t)]$$

where

$$\beta_p = k_p A_n$$

The frequency deviation of the single-tone PM wave is

$$s_{FM}(t) = A_c \cos[2\pi f_c t + \beta_f \sin(2\pi f_m t)]$$

Spectral Analysis of Single-Tone FM Wave

$$s_{FM}(t) = \mathrm{Re}\{A_c e^{j2\pi f_c t} e^{j\beta \sin(2\pi f_n t)}\}$$

$$e^{j\beta \sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_m t}$$

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} e^{j\beta \sin(2\pi f_m t)} e^{-j2\pi n f_m t} dt$$

Let $2\pi f_m t = x$, then Eq. (5.24) reduces to

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\beta \sin(x)} e^{-jnx} dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta \sin(x) - nx)} dx$$

The integral on the right-hand side is known as the n^th order Bessel function of the first kind and is denoted by $J_n(\beta)$. Therefore, $c_n = J_n(\beta)$ and Eq. (4.23) can be written as

$$e^{j\beta \sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t} \tag{5.26}$$

$$s_{FM}(t) = \mathrm{Re}\left\{A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t}\, e^{j2\pi f_c t}\right\}$$
$$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + n f_m)t] \tag{5.27}$$

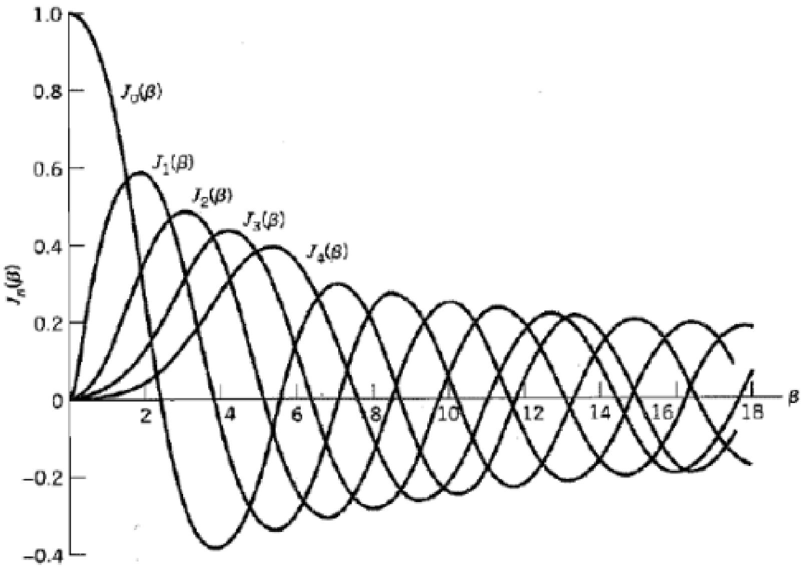
Taking Fourier transform of Eq. (5.27), we get

$$S(f) = \frac{1}{2}A_c \sum_{n=-\infty}^{\infty} J_n(\beta)[\delta(f - f_c - n f_m) + \delta(f + f_c + n f_m)] \tag{5.28}$$

The Bessel function may be expanded in a power series given by

$$J_n(\beta) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{1}{2}\beta\right)^{n+2k}}{k!\,(k+n)!}$$

Plots of Bessel function $J_n(\beta)$ versus modulation index β for $n = 0, 1, 2, 3, 4$ are shown in Figure 5.3.



One property of Bessel function is that

$$J_{-n}(\beta) = \begin{cases} J_n(\beta), & n \text{ even} \\ -J_n(\beta), & n \text{ odd} \end{cases}$$

One more property of Bessel function is that

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

The average power of the FM wave remains constant. To prove this, let us determine the average power of Eq. (5.27) which is equal to

$$P = \frac{1}{2} A_c^2 \sum_{n=-\infty}^{\infty} J_n^2(\beta)$$

Using Eq. (5.31), the average power P is now

$$P = \frac{1}{2} A_c^2$$

For single-tone frequency modulation, the approximated bandwidth is determined by

$$B = 2(\Delta f + f_m) = 2(\beta + 1)f_m = 2\Delta f\left(1 + \frac{1}{\beta}\right)$$

This expression is regarded as the Carson's rule. The FM bandwidth determined by this rule accommodates at least 98 % of the total power.

For an arbitrary message signal n(t) with bandwidth or maximum frequency W, the bandwidth of the corresponding FM wave may be determined by Carson's rule as

$$B = 2(\Delta f + W) = 2(D + 1)W = 2\Delta f\left(1 + \frac{1}{D}\right)$$

GENERATION OF FM WAVES

Indirect Method (Armstrong Method) of FM Generation

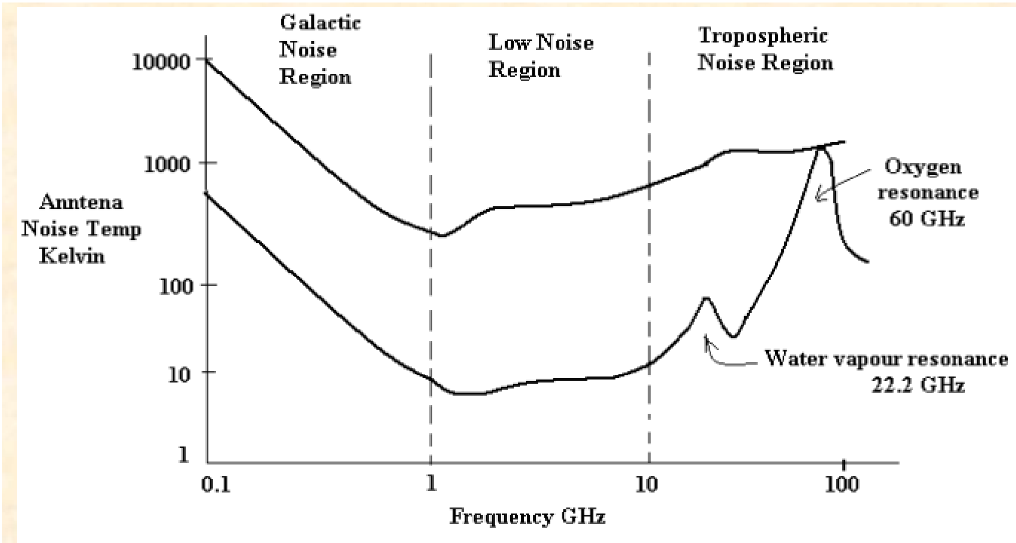
٦. Noise

Noise is a general term which is used to describe an unwanted signal which affects a wanted signal. These unwanted signals arise from a variety of sources which may be considered in one of two main categories:-

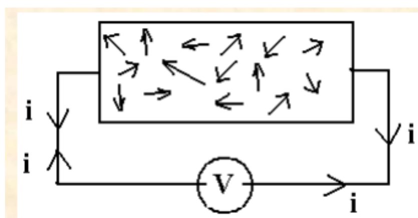
- *Interference, usually from a human source (man made)*
- *Naturally occurring random noise*

Natural Noise

Naturally occurring external noise sources include atmosphere disturbance (e.g. electric storms, lighting, ionospheric effect etc), so called 'Sky Noise' or Cosmic noise which includes noise from galaxy, solar noise and 'hot spot'due to oxygen and water vapour resonance in the earth's atmosphere.



Thermal Noise (Johnson)



This type of noise is generated by all resistances (e.g. a resistor, semiconductor, the resistance of a resonant circuit, i.e. the real part of the impedance, cable etc).

Experimental results (by Johnson) and theoretical studies (by Nyquist) give the mean square noise voltage as

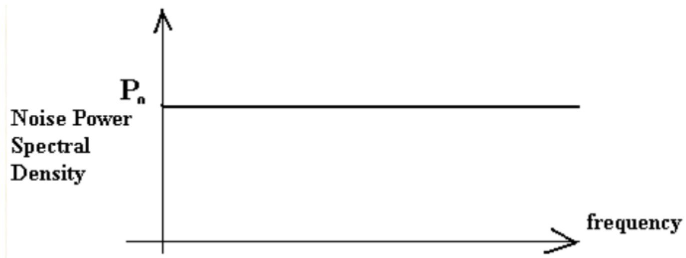
$$\bar{V}^2 = 4kTBR \ (volt^2)$$

k = Boltzmann's constant = 1.38 x 10^{-23} Joules per K
T = absolute temperature
B = bandwidth noise measured in (Hz)
R = resistance (ohms)

relating noise power, N, to the temperature and bandwidth

N = k TB watts

Thermal noise is often referred to as 'white noise' because it has a uniform 'spectral density'.



Noise Evaluation & Signal to Noise Ratio

$$\left(\frac{S}{N}\right)_{ratio} = \frac{S}{N}$$

$$\left(\frac{S}{N}\right)_{dB} = 10\log_{10}\left(\frac{S}{N}\right)$$

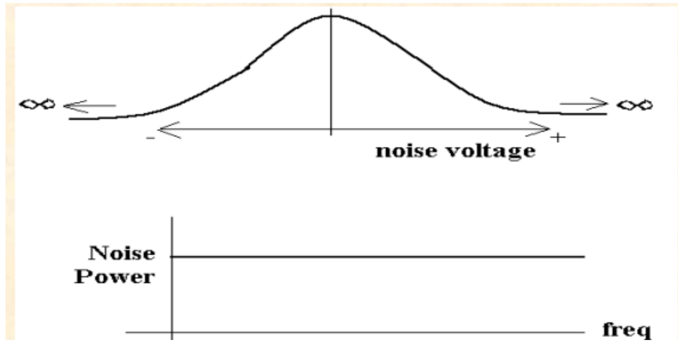
Also recall that

$$S_{dBm} = 10\log_{10}\left(\frac{S(mW)}{1mW}\right)$$

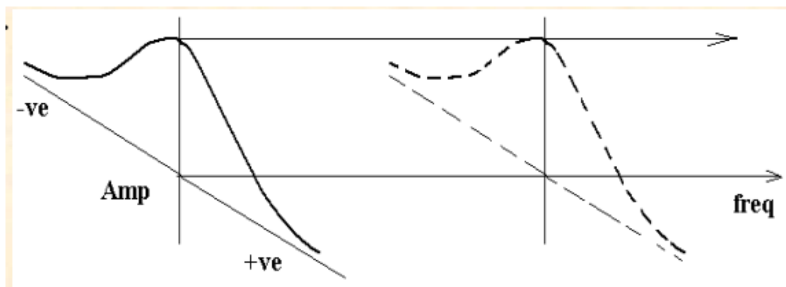
$$and\ \ N_{dBm} = 10\log_{10}\left(\frac{N(mW)}{1mW}\right)$$

$$i.e.\ \left(\frac{S}{N}\right)_{dB} = 10\log_{10} S - 10\log_{10} N$$

$$\left(\frac{S}{N}\right)_{dB} = S_{dBm} - N_{dBm}$$



The probability of amplitude of noise at any frequency or in any band of frequencies (e.g. 1 Hz, 10Hz… 100 KHz .etc) is a Gaussian

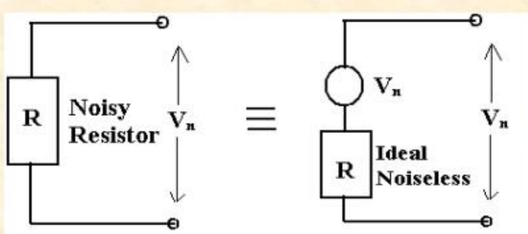


Noise may be quantified in terms of noise power spectral density, p_o watts per Hz, from which Noise power N may be expressed as

$$N = p_o B_n \text{ watts}$$

Analysis of Noise In Communication Systems

This thermal noise may be represented by an equivalent circuit as shown below



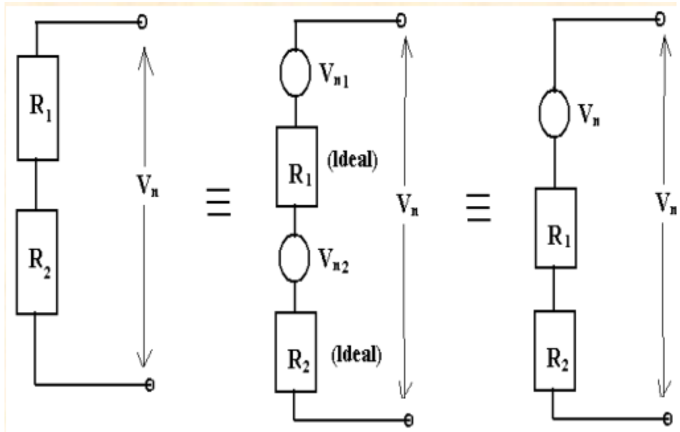
$$\overline{V^2} = 4kTBR \ (volt^2)$$

(mean square value , power)

then $V_{RMS} = \sqrt{\overline{V^2}} = 2\sqrt{kTBR} = V_n$

i.e. V_n is the RMS noise voltage.

Resistors in Series



Assume that R_1 at temperature T_1 and R_2 at temperature T_2, then

$$\overline{V_n^2} = \overline{V_{n1}^2} + \overline{V_{n2}^2}$$

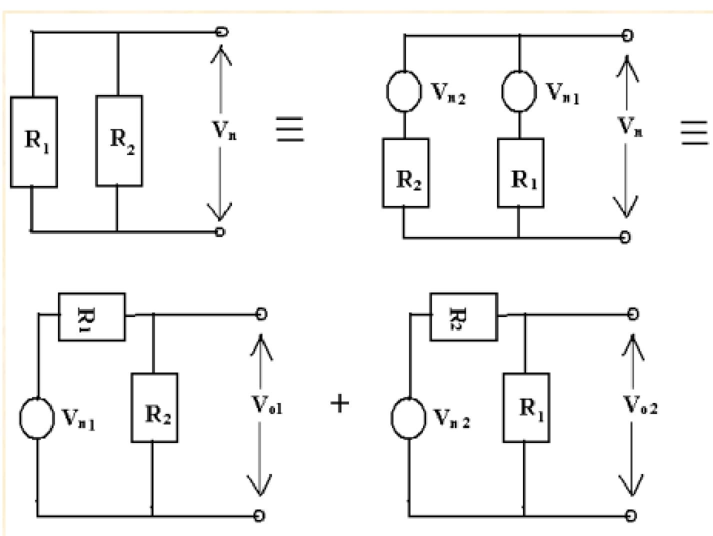
$$\overline{V_{n1}^2} = 4kT_1 B R_1$$

$$\overline{V_{n2}^2} = 4kT_2 B R_2$$

$$\overline{V_n^2} = 4kB(T_1 R_1 + T_2 R_2)$$

$$\overline{V_n^2} = 4kTB(R_1 + R_2)$$

Resistance in Parallel



$$V_{o1} = V_{n1}\frac{R_2}{R_1 + R_2} \qquad V_{o2} = V_{n2}\frac{R_1}{R_1 + R_2}$$

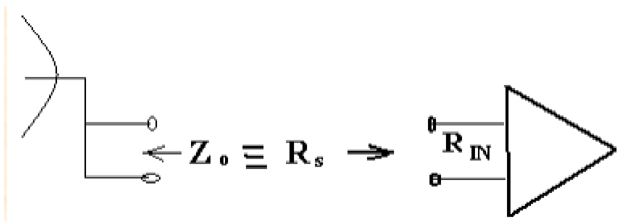
$$\overline{V_n^{\,2}} = \overline{V_{o1}^{\,2}} + \overline{V_{o2}^{\,2}}$$

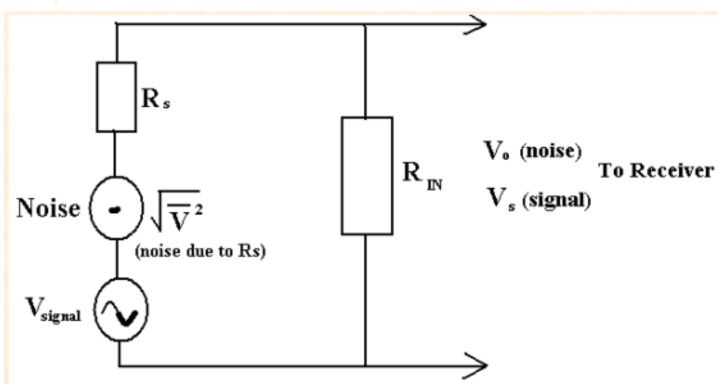
$$\overline{V_n^{\,2}} = \frac{4kB}{(R_1 + R_2)^2}\left[R_2^2\, T_1 R_1 + R_1^2\, T_2 R_2\right] \times \left(\frac{R_1 R_2}{R_1 R_2}\right)$$

$$\overline{V_n^{\,2}} = \frac{4kB R_1 R_2 (T_1 R_1 + T_2 R_2)}{(R_1 + R_2)^2}$$

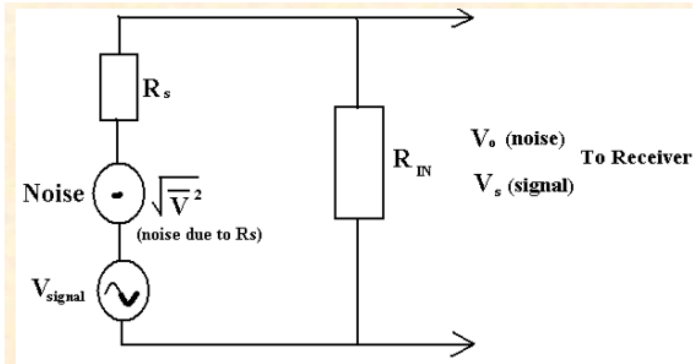
$$\overline{V_n^{\,2}} = 4kTB\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

Matched Communication Systems





An equivalent circuit, when the line is connected to the receiver is shown below.



The RMS voltage output, V$_o$ (noise) is

$$V_o\,(noise) = \sqrt{\overline{v^2}}\left(\frac{R_{IN}}{R_{IN}+R_S}\right)$$

Similarly, the signal voltage output due to V$_{signal}$ at input is

$$Vs(_{signal}) = (V_{signal})\left(\frac{R_{IN}}{R_{IN}+R_s}\right)$$

For maximum power transfer, the input R_{IN} is matched to the source R_S, i.e. R_{IN} $= R_S$ = R (say)

$$V_o\,(noise) = \sqrt{\overline{v^2}}\left(\frac{R}{2R}\right) = \frac{\sqrt{\overline{v^2}}}{2}\ (RMS\ \ Value)$$

Signal to Noise

$$\frac{S}{N} = \frac{Signal\ Power}{Noise\ Power}$$

$$\left(\frac{S}{N}\right)_{dB} = 10\log_{10}\left(\frac{S}{N}\right)$$

$$\left(\frac{S}{N}\right)_{dB} = S_{dBm} - N_{dBm}$$ for S and N measured in mW.

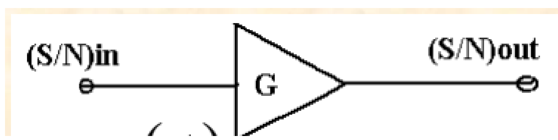
Noise Factor- Noise Figure

$$\text{Noise factor } F = \frac{\left(S/N\right)_{IN}}{\left(S/N\right)_{OUT}}$$

•F equals to 1 for noiseless network and in general F > 1. The noise figure in the noise factor quoted in dB
i.e. Noise Figure F dB = 10 log10 F F ≥ 0 dB

•The noise figure / factor is the measure of how much a network degrades the (S/N)IN, the lower the value of F, the better the network.

Noise Factor for Active Elements

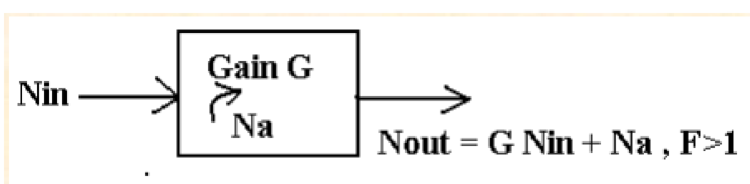


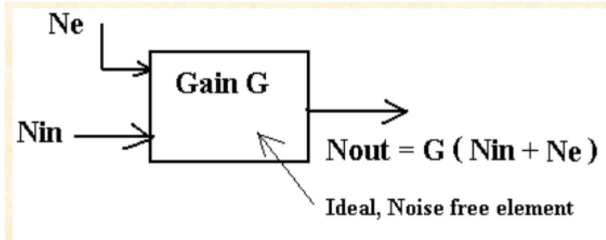
$$F = \frac{\left(S/N\right)_{IN}}{\left(S/N\right)_{OUT}} = \frac{S_{IN}}{N_{IN}} \frac{N_{OUT}}{S_{OUT}} \qquad \text{But} \qquad S_{OUT} = G\, S_{IN}$$

Therefore

$$F = \frac{S_{IN}}{N_{IN}} \frac{N_{OUT}}{G\, S_{IN}} = \frac{N_{OUT}}{G\, N_{IN}}$$

N_{OUT} is increased by noise due to the active element i.e.





$$\text{Hence } F = \frac{N_{OUT}}{G\,N_{IN}} = F = \frac{G(N_{IN}+N_e)}{G\,N_{IN}}$$

Rearranging gives,

$$N_e = (F-1)\,N_{IN}$$

Noise Temperature

N_{IN} is the 'external' noise from the source i.e. $N_{IN} = kT_S\, B_n$

T_S is the equivalent noise temperature of the source (usually 290K).

We may also write $N_e = kT_e\, B_n$, where T_e is the equivalent noise temperature of the element i.e. with noise factor F and with source temperature T_S.

i.e. $kT_e\, B_n = (F\text{-}1)\,kT_S\, B_n$

or $T_e = (F\text{-}1)T_S$

Noise Factor for Passive Elements

Since $F = \dfrac{S_{IN}}{N_{IN}}\dfrac{N_{OUT}}{S_{OUT}}$ and $N_{OUT} = N_{IN}$.

$$F = \frac{S_{IN}}{G\,S_{IN}} = \frac{1}{G}$$

If we let L denote the insertion loss (ratio) of the network i.e. insertion loss

$$L_{dB} = 10 \log L$$

Then

$$L = \frac{1}{G} \text{ and hence for passive network}$$

$$F = L$$

Also, since $T_e = (F\text{-}1)T_S$

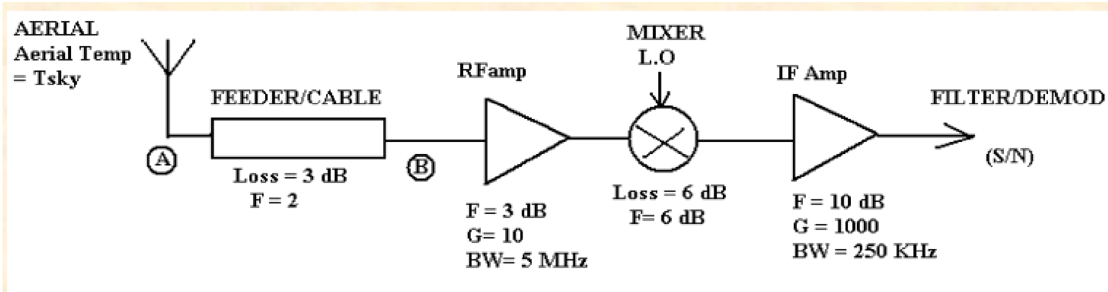
Then for passive network

$$T_e = (L\text{-}1)T_S$$

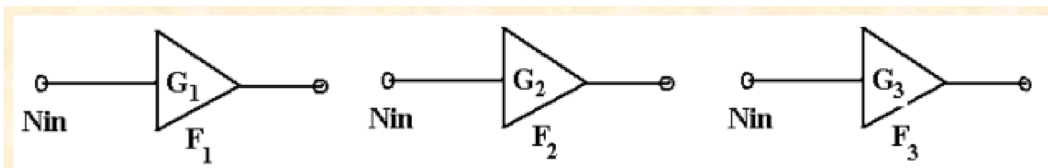
Typical values of noise temperature, noise figure and gain

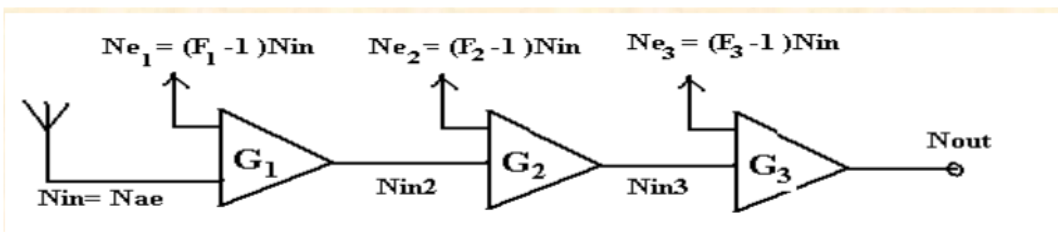
Device	**Frequency**	T_e **(K)**	F_{dB} **(dB)**	**Gain (dB)**
Maser Amplifier	9 GHz	4	0.06	20
Ga As Fet amp	9 GHz	330	303	6
Ga As Fet amp	1 GHz	110	1.4	12
Silicon Transistor	400 MHz	420	3.9	13
L C Amp	10 MHz	1160	7.0	50
Type N cable	1 GHz		2.0	2.0

Cascaded Network



System Noise Figure





$$N_{OUT} = G_3\left(N_{IN3} + N_{e3}\right)$$
$$= G_3\left(N_{IN3} + (F_3 - 1)N_{IN}\right)$$

$$N_{IN3} = G_2\left(N_{IN2} + N_{e2}\right) = G_2\left(N_{IN2} + (F_2 - 1)N_{IN}\right)$$

$$N_{IN2} = G_1\left(N_{ae} + (F_1 - 1)N_{IN}\right)$$

The overall system Noise Factor is

$$F_{sys} = \frac{N_{OUT}}{GN_{IN}} = \frac{N_{OUT}}{G_1G_2G_3N_{ae}}$$
$$= 1 + (F_1 - 1)\frac{N_{IN}}{N_{ae}} + \frac{(F_2 - 1)}{G_1}\frac{N_{IN}}{N_{ae}} + \frac{(F_3 - 1)}{G_1G_2}\frac{N_{IN}}{N_{ae}}$$

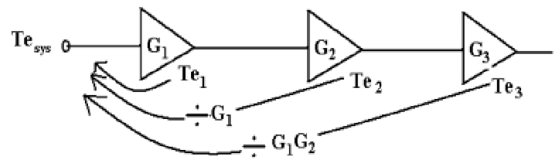
then for n elements in cascade.

$$F_{sys} = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1G_2} + \frac{(F_4 - 1)}{G_1G_2G_3} + \dots\dots\dots + \frac{(F_n - 1)}{G_1G_2\dots\dots\dots G_{n-1}}$$

The equation is called FRIIS Formula.

Since $T_e = (L-1)T_S$, i.e. $F = 1 + \frac{T_e}{T_s}$

Then



$$F_{sys} = 1 + \frac{T_{e\,sys}}{T_s}$$

$$\begin{cases} where\, T_{e\ sys}\ is\ the\ equivalent\ Noise\ temperature\ of\ the\ system \\ and\, T_s\ is\ the\ noise\ temperature\ of\ the\ source \end{cases}$$

$$i.e.\ from\, F_{sys} = F_1 + \frac{(F_2 - 1)}{G_1} + ...etc$$

which gives

$$T_{e\,sys} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \frac{T_{e4}}{G_1 G_2 G_3} + ...$$